

EXACT SOLUTIONS OF AN UNSTEADY CONDUCTING DUSTY FLUID FLOW BETWEEN NON-TORSIONAL OSCILLATING PLATE AND A LONG WAVY WALL

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Abstract. *Frenet frames are a central construction in modern differential geometry, in which the structure is described with respect to an object of interest rather than with respect to external coordinate systems. In the present paper we have study the geometry of laminar flow of an incompressible viscous unsteady MHD dusty fluid with uniform distribution of dust particles between a parallel flat wall and a long wavy wall in Frenet frame field system. The flow is due to the presence of a uniform transverse magnetic field, non-torsional oscillations of the plate and time dependent pressure gradient. Velocities of both fluid and dust phases are obtained using Laplace transform technique. Further the shear stress (skin friction) is obtained at the boundaries*

Keywords: *Frenet frame field system, laminar flow, dusty fluid, velocity of dust phase and fluid phase, motion for a finite time.*

1. INTRODUCTION

The flow of an electrically conducting dusty fluid through a channel in the presence of a transverse magnetic field is encountered in a variety of applications such as magneto hydrodynamic (MHD) generators, pumps, accelerators and flow meters, wastewater treatment, power plant piping, purification of the crude oils, combustion and petroleum transport. P.G Saffman [21] has discussed the stability of the laminar flow of a dusty gas in which the dust particles are uniformly distributed. The study of dusty viscous fluid under the influence of different physical conditions has been carried out by several authors; Nag and Jana [10] have studied unsteady Couette flow of a dusty gas between two infinite parallel plates, when one plate of the channel is kept stationary and other plate moves uniformly in its own plane. Dalal [6] analyzed the generalized Couette flow of dusty gas due to an impulsive pressure gradient as well as due to impulsive start of the lower plate. Singh and Singh [22] studied the laminar convective flow of an incompressible, conducting viscous fluid embedded with non-conducting dust particles through a vertical parallel plate channel in the presence of uniform magnetic field and constant pressure gradient taking volume fraction of the particles into consideration when one plate of the channel is fixed and the other is oscillating in time and in magnitude about a constant non-zero mean. Attia [1, 2] studied the effects of variable

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viscosity on the unsteady flow of an electrically conducting, viscous, incompressible dusty fluid and heat transfer between parallel non-conducting porous plates when a uniform magnetic field is applied perpendicular to the plates. Liu [13] has studied the flow induced by an oscillating infinite flat plate in a dusty gas. Michael and Miller [16] investigated the motion of dusty gas with uniform distribution of the dust particles placed in the semi-infinite space above a rigid plane boundary. Later, Samba Siva Rao [11] has obtained the analytical solutions for the dusty fluid flow through a circular tube under the influence of constant pressure gradient, using appropriate boundary conditions. Ong and Nicholls [25] have extended the problem to cover the case of flow near an infinite wall which executes simple harmonic motion parallel to itself. Later, M.C.Baral [05] has discussed the plane parallel flow of conducting dusty gas, A.Eric et.al. [07] has studied the quantitative assessment of steady and pulsatile flow fields in a parallel plate flow chamber. Thierry Feraille et.al., [14] discussed the channel flow induced by wall injection Unsteady Flow Between a Non-torsional Oscillating Plate and a Long Wavy Wall of fluid and particles. Mitra and Bhattacharyya [15] studied unsteady hydromagnetic laminar flow of a conducting dusty fluid between two parallel plates started impulsively from rest.

During the second part of the 20th century, some researchers like Kanwal [12], Truesdell [23], Indrasena [11], Purushotham [19], Bagewadi and Giresha [2, 3,5] have applied differential geometry techniques to investigate the kinematical properties of fluid flows in the field of fluid mechanics. Further, recently the authors [6,7,14,15] have studied two-dimensional dusty fluid flow in Frenet frame field system. This work is focused on the mathematical modeling of the flow of an electrically conducting viscous incompressible fluid which suspended non-conducting small spherical dust particles between a non-torsional oscillating plate and a long wavy wall. The flow is due to the presence of a uniform transverse magnetic field, non-torsional oscillations of the plate and time dependent pressure gradient. Initially it is assumed that both the conducting fluid and the non-conducting dust particles are to be at rest. Applying Laplace transform technique, the velocity fields for fluid and dust particles have been obtained. Also the skin friction at both the walls has been calculated. Finally the graphs are plotted for different values of Hartmann number.

2. EQUATIONS OF MOTION

The equations of motion of an unsteady viscous incompressible fluid with uniform distribution of dust particles in a porous medium are given by [17]:

For Fluid Phase

$$\nabla \cdot \vec{u} = 0 \quad (\text{Continuity}) \quad (1)$$

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla) \vec{u} = -\rho^{-1} \nabla p + \nu \nabla^2 \vec{u} + \frac{kN}{\rho} (\vec{v} - \vec{u}) - \frac{1}{\rho} (\vec{J} \times \vec{B}) \quad (\text{Linear Momentum}) \quad (2)$$

For Dust Phase

$$\nabla \cdot \vec{v} = 0 \quad (\text{Continuity}) \quad (3)$$

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} = \frac{k}{m} (\vec{u} - \vec{v}) \quad (\text{Linear Momentum}) \quad (4)$$

where:

- \vec{u} - velocity of the fluid phase,
- \vec{v} - velocity of dust phase,
- ρ - density of the gas, p -pressure of the fluid,
- N - number density of dust particles,
- ν - kinematic viscosity,
- $k = 6\pi a\mu$ - Stoke's resistance (drag coefficient),
- m - mass of the dust particle,
- t - time,
- μ - the co-efficient of viscosity of fluid particles.

Let \vec{s} , \vec{n} , \vec{b} be triply orthogonal unit vectors along tangent, principal normal, binormal respectively to the spatial curves of congruence's formed by fluid phase velocity and dusty phase velocity lines respectively as shown in the Fig. 1.

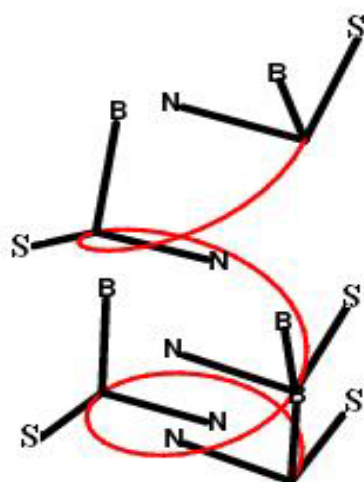


Fig. 1. Frenet Frame Field System.

Geometrical relations are given by Frenet Formulae [19]

- i) $\frac{\partial \vec{s}}{\partial s} = k_s \vec{n}, \frac{\partial \vec{n}}{\partial s} = \tau_s \vec{b} - k_s \vec{s}, \frac{\partial \vec{b}}{\partial s} = -\tau_s \vec{n}$
- ii) $\frac{\partial \vec{n}}{\partial n} = k'_n \vec{s}, \frac{\partial \vec{b}}{\partial n} = -\sigma'_n \vec{s}, \frac{\partial \vec{s}}{\partial n} = \sigma'_n \vec{b} - k'_n \vec{n}$ (5)
- iii) $\frac{\partial \vec{b}}{\partial b} = k''_b \vec{s}, \frac{\partial \vec{n}}{\partial b} = -\sigma''_b \vec{s}, \frac{\partial \vec{s}}{\partial b} = \sigma''_b \vec{n} - k''_b \vec{b}$
- iv) $\nabla \cdot \vec{s} = \theta_{ns} + \theta_{bs}, \nabla \cdot \vec{n} = \theta_{bs} - k_s, \nabla \cdot \vec{b} = \theta_{nb}$

where $\partial/\partial s$, $\partial/\partial n$ and $\partial/\partial b$ are the intrinsic differential operators along fluid phase velocity (or dust phase velocity) lines along tangential, principal normal and binormal respectively. The functions (k_s, K'_n, K''_b) and $(\tau_s, \sigma'_n, \sigma''_b)$ are the curvatures and torsion of the above curves and θ_{ns} and θ_{bs} are normal deformations of these spatial curves along their principal normal and binormal respectively.

3. FORMULATION OF THE PROBLEM

Let the flow of an unsteady viscous incompressible, dusty fluid between a non-torsional oscillating plate and a long wavy wall as shown in the figure-2. The number density of the dust particles is taken as a constant throughout the flow. It is assumed that the dust particles are electrically non conducting and neutral. The motion of the dusty fluid is due to magnetic field of uniform strength B_0 , non-torsional oscillations of the plate and under the influence of time dependent pressure gradient. Under these assumptions the flow will be a parallel flow in which the streamlines are along the tangential direction and the velocities are varies along binormal direction and with time t , since we extended the fluid to infinity in the principal normal direction.

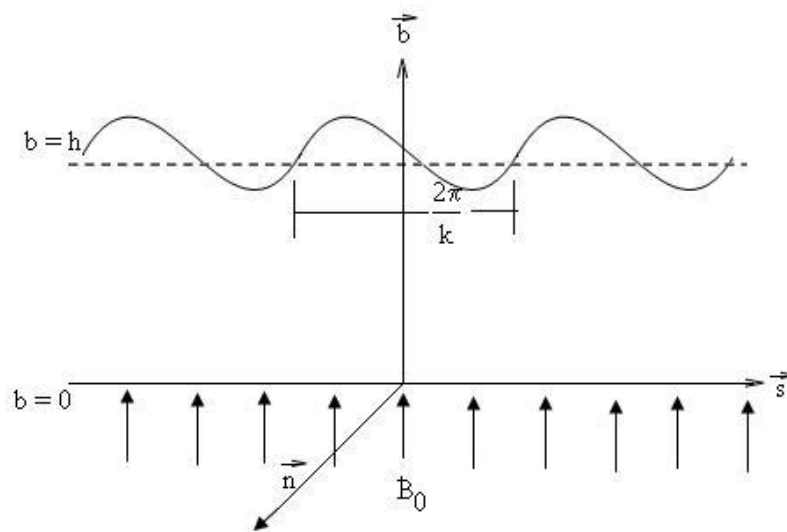


Fig. 2. Geometry of the flow pattern.

For the above described flow the velocities of fluid and dust are of the form

$$\vec{u} = u_s \vec{s}, \quad \vec{v} = v_s \vec{s} \quad (6)$$

i.e., $u_b = u_n = 0$ and $v_b = v_n = 0$, where (u_s, u_n, u_b) and (v_s, v_n, v_b) denote the velocity components of fluid and dust respectively.

4. SOLUTION OF THE PROBLEM

By virtue of system of equations (5) the intrinsic decomposition of equations (2) and (4) give the following forms;

$$\frac{\partial u_s}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial s} + \nu \left[\frac{\partial^2 u_s}{\partial b^2} - C_r u_s \right] + \frac{kN}{\rho} (v_s - u_s) - D u_s \quad (7)$$

$$2u_s^2 k_s = -\frac{1}{\rho} \frac{\partial p}{\partial n} + \nu \left[2\sigma_b'' \frac{\partial u_s}{\partial b} - u_s k_s^2 \right] \quad (8)$$

$$0 = -\frac{1}{\rho} \frac{\partial p}{\partial b} + \nu \left[u_s k_s \tau_s - 2k_b'' \frac{\partial u_s}{\partial b} \right] \quad (9)$$

$$\frac{\partial v_s}{\partial t} = \frac{k}{m} (u_s - v_s) \quad (10)$$

$$v_s^2 k_s = 0 \quad (11)$$

where $C_r = (\sigma_n'^2 + \pi_s'^2 + k_b'^2)$ is called curvature number [18].

From equation (11) we see that either $v_s = 0$ or $k_s = 0$. The choice $v_s = 0$ is impossible, since if it happens then $u_s = 0$, which shows that the flow doesn't exist. Hence $k_s = 0$, it suggests that the curvature of the streamline along tangent direction is zero. Thus no radial flow exists.

Equation (7) and (10) are to be solved subject to the initial and boundary conditions.

Initial condition:

$$at t = 0.; u_s = 0, v_s = 0$$

Boundary conditions:

$$for t > 0 ; u_s = a_1 e^{i\omega_1 t} + a_2 e^{i\omega_2 t}, at b = 0$$

$$and u_s = 0, b = \alpha^* \cos(\beta^* s)$$

where α^* is the amplitude parameter, β^* is the frequency parameter and $a_1, a_2, \omega_1, \omega_2$ are constants.

Let us consider the following non-dimensional quantities.

$$u_s^* = \frac{u_s h}{U}, v_s^* = \frac{v_s h}{U}, b^* = \frac{b}{h}, t^* = \frac{tU}{h^2}, p^* = \frac{ph^2}{\rho U^2}, s^* = \frac{s}{h}$$

Let $\wp(t)$ be the time dependent pressure gradient to be impressed on the system for $t > 0$. So we can write

$$-\frac{\partial p}{\partial s} = \wp(t).$$

We define Laplace transforms of U_s and V_s as

$$U_s = \int_0^\infty e^{-xt} u_s dt \text{ and } V_s = \int_0^\infty e^{-xt} v_s dt \tag{12}$$

Applying the Laplace transform to equations (7) and (10) and to boundary conditions, then by using initial conditions one obtains

$$xU_s = F(X) + \frac{h}{R_e} \frac{d^2 U_s}{db^2} - \frac{h^3 C_r}{R_e} U_s + \frac{h^2 l}{U\tau} (U_s - V_s) - MU_s \tag{13}$$

$$xV_s = \frac{h^2}{U\tau} (U_s - V_s) \tag{14}$$

$$U_s = \frac{a_1}{x - i\sigma_1} + \frac{a_2}{x - i\sigma_2} \text{ at } b = 0 \text{ and } U_s = 0 \text{ at } b = \alpha \cos(\beta s) \tag{15}$$

where $l = \frac{mN}{\rho}$, $\tau = \frac{m}{k}$ and $F(X)$ is Laplace transform of $\wp(t)$.

Equation (13) implies

$$V_s = \frac{h^2}{(h^2 + xU\tau)} U_s \tag{16}$$

Eliminating V_s from (13) and (14) we obtain the following equation

$$\frac{d^2 U_s}{db^2} - Q^2 U_s = -\frac{R_e}{h} F(x) \tag{17}$$

where $Q^2 = h^2 C_r + \frac{MR_e}{h} + \frac{xR_e}{h} \left(1 + \frac{lh^2}{(h^2 + xU\tau)} \right)$.

5. MOTION FOR A FINITE TIME

In this case we take $-\frac{\partial p}{\partial s} = P_0 [H(t) - H(t-T)]$, where P_0 and d are constants and $H(t)$ is the Heaviside unit step function. We can obtain U_s by solving equation (17) and V_s by using U_s and equations (16). By taking inverse Laplace transform the expression for both fluid and dust phase velocities are obtained as

$$\begin{aligned}
 u_s = & \frac{2P_0}{\pi} \sum_{r=0}^{\infty} \left[\frac{1 - (-1)^r}{r} \right] \sin \left(\frac{r\pi b}{\alpha \cos(\beta s)} \right) \\
 & \times \left[\frac{e^{x_1 t} (1 - e^{T x_1}) (h^2 + x_1 U \tau)^2}{x_1 \left[(h^2 + x_1 U \tau)^2 + lh^4 \right]} + \frac{e^{x_2 t} (1 - e^{T x_2}) (h^2 + x_2 U \tau)^2}{x_2 \left[(h^2 + x_2 U \tau)^2 + lh^4 \right]} \right] \\
 & - \left[\frac{a_1 \left[(\Psi_1 \cos(\sigma_1 t) + \Psi_2 \sin(\sigma_1 t)) - i(\Psi_2 \cos(\sigma_1 t) - \Psi_1 \sin(\sigma_1 t)) \right]}{(C^2 + D^2)} \right] \\
 & + \frac{2a_1 h \pi}{R_e \alpha^2 \cos^2(\beta s)} \sum_{r=0}^{\infty} r \sin \left(\frac{r\pi b}{\alpha \cos(\beta s)} \right) \\
 & \times \left[\frac{e^{x_1 t} (x_1 + i\sigma_1) (h^2 + x_1 U \tau)^2}{(x_1^2 + \sigma_1^2) \left[(h^2 + x_1 U \tau)^2 + lh^4 \right]} + \frac{e^{x_2 t} (x_2 + i\sigma_1) (h^2 + x_2 U \tau)^2}{(x_2^2 + \sigma_1^2) \left[(h^2 + x_2 U \tau)^2 + lh^4 \right]} \right] \\
 & - \left[\frac{a_2 \left[(\Phi_1 \cos(\sigma_2 t) + \Phi_2 \sin(\sigma_2 t)) + i(\Phi_2 \cos(\sigma_2 t) - \Phi_1 \sin(\sigma_2 t)) \right]}{(G^2 + H^2)} \right] \\
 & + \frac{2a_2 h \pi}{R_e \alpha^2 \cos^2(\beta s)} \sum_{r=0}^{\infty} r \sin \left(\frac{r\pi b}{\alpha \cos(\beta s)} \right) \\
 & \times \left[\frac{e^{x_1 t} (x_1 - i\sigma_2) (h^2 + x_1 U \tau)^2}{(x_1^2 + \sigma_2^2) \left[(h^2 + x_1 U \tau)^2 + lh^4 \right]} + \frac{e^{x_2 t} (x_2 - i\sigma_2) (h^2 + x_2 U \tau)^2}{(x_2^2 + \sigma_2^2) \left[(h^2 + x_2 U \tau)^2 + lh^4 \right]} \right] \\
 v_s = & \frac{2P_0 h^2}{\pi} \sum_{r=0}^{\infty} \left[\frac{1 - (-1)^r}{r} \right] \sin \left(\frac{r\pi b}{\alpha \cos(\beta s)} \right) \\
 & \times \left[\frac{e^{x_1 t} (h^2 + x_1 U \tau)^2}{\left[(h^2 + x_1 U \tau)^2 + lh^4 \right]} + \frac{e^{x_2 t} (h^2 + x_2 U \tau)^2}{\left[(h^2 + x_2 U \tau)^2 + lh^4 \right]} \right]
 \end{aligned}$$

$$\begin{aligned}
 & - \left[\frac{a_1 h^2 \left[(\Psi_3 \cos(\sigma_1 t) + \Psi_4 \sin(\sigma_1 t)) - i(\Psi_4 \cos(\sigma_1 t) - \Psi_3 \sin(\sigma_1 t)) \right]}{(C^2 + D^2)(h^4 + \sigma_2^2 U^2 \tau^2)} \right] \\
 & + \frac{2a_1 h^3 \pi}{R_e \alpha^2 \cos^2(\beta s)} \sum_{r=0}^{\infty} r \sin\left(\frac{r\pi b}{\alpha \cos(\beta s)}\right) \\
 & \times \left[\frac{e^{x_1 t} (x_1 + i\sigma_1)(h^2 + x_1 U \tau)^2}{(x_1^2 + \sigma_1^2)[(h^2 + x_1 U \tau)^2 + lh^4]} + \frac{e^{x_2 t} (x_2 + i\sigma_1)(h^2 + x_2 U \tau)^2}{(x_2^2 + \sigma_1^2)[(h^2 + x_2 U \tau)^2 + lh^4]} \right] \\
 & - \left[\frac{a_2 h^2 \left[(\Phi_3 \cos(\sigma_2 t) + \Phi_4 \sin(\sigma_2 t)) + i(\Phi_4 \cos(\sigma_2 t) - \Phi_3 \sin(\sigma_2 t)) \right]}{(G^2 + H^2)(h^4 + \sigma_2^2 U^2 \tau^2)} \right] \\
 & + \frac{2a_2 h^3 \pi}{R_e \alpha^2 \cos^2(\beta s)} \sum_{r=0}^{\infty} r \sin\left(\frac{r\pi b}{\alpha \cos(\beta s)}\right) \\
 & \times \left[\frac{e^{x_1 t} (x_1 - i\sigma_2)(h^2 + x_1 U \tau)^2}{(x_1^2 + \sigma_2^2)[(h^2 + x_1 U \tau)^2 + lh^4]} + \frac{e^{x_2 t} (x_2 - i\sigma_2)(h^2 + x_2 U \tau)^2}{(x_2^2 + \sigma_2^2)[(h^2 + x_2 U \tau)^2 + lh^4]} \right]
 \end{aligned}$$

6. SHEARING STRESS (SKIN FRICTION)

The Shear stress at the boundaries $b = 0$ and $b = \alpha \cos \beta s$ are given by

$$\begin{aligned}
 D_0 = & \frac{2P_0 \mu}{\alpha \cos(\beta s)} \sum_{r=0}^{\infty} \left[1 - (-1)^r \right] \\
 & \times \left[\frac{e^{x_1 t} (1 - e^{-Tx_1})(h^2 + x_1 U \tau)^2}{\left[x_1 (h^2 + x_1 U \tau)^2 + lh^4 \right]} + \frac{e^{x_2 t} (1 - e^{-Tx_2})(h^2 + x_2 U \tau)^2}{\left[x_2 (h^2 + x_2 U \tau)^2 + lh^4 \right]} \right] \\
 & - \left[\frac{a_1 \mu \left[(L_1 \cos(\sigma_2 t) - L_2 \sin(\sigma_2 t)) + i(L_2 \cos(\sigma_2 t) + L_1 \sin(\sigma_2 t)) \right]}{(C^2 + D^2)} \right] \\
 & + \frac{2\mu a_1 h \pi^2}{R_e \alpha^3 \cos^3(\beta s)} \sum_{r=0}^{\infty} r^2 \\
 & \times \left[\frac{e^{x_1 t} (x_1 + i\sigma_2)(h^2 + x_1 U \tau)^2}{(x_1^2 + \sigma_2^2)[(h^2 + x_1 U \tau)^2 + lh^4]} + \frac{e^{x_2 t} (x_2 + i\sigma_2)(h^2 + x_2 U \tau)^2}{(x_2^2 + \sigma_2^2)[(h^2 + x_2 U \tau)^2 + lh^4]} \right] \\
 & - \left[\frac{a_2 \mu \left[(L_3 \cos(\sigma_2 t) - L_4 \sin(\sigma_2 t)) - i(L_4 \cos(\sigma_2 t) + L_3 \sin(\sigma_2 t)) \right]}{(G^2 + H^2)} \right] \\
 & + \frac{2\mu a_2 h \pi^2}{R_e \alpha^3 \cos^3(\beta s)} \sum_{r=0}^{\infty} r^2 \\
 & \times \left[\frac{e^{x_1 t} (x_1 - i\sigma_2)(h^2 + x_1 U \tau)^2}{(x_1^2 + \sigma_2^2)[(h^2 + x_1 U \tau)^2 + lh^4]} + \frac{e^{x_2 t} (x_2 - i\sigma_2)(h^2 + x_2 U \tau)^2}{(x_2^2 + \sigma_2^2)[(h^2 + x_2 U \tau)^2 + lh^4]} \right]
 \end{aligned}$$

$$\begin{aligned}
D_1 = & \frac{2P_0\mu}{\alpha \cos(\beta s)} \sum_{r=0}^{\infty} [(-1)^r - 1] \\
& \times \left[\frac{e^{x_1 t} (1 - e^{-Tx_1})(h^2 + x_1 U\tau)^2}{x_1 [(h^2 + x_1 U\tau)^2 + lh^4]} + \frac{(1 - e^{-Tx_2})e^{x_2 t} (h^2 + x_2 U\tau)^2}{x_2 [(h^2 + x_2 U\tau)^2 + lh^4]} \right] \\
& - \left[\frac{a_1 \mu [(M_1 \cos(\sigma_1 t) + M_2 \sin(\sigma_1 t)) - i(M_2 \cos(\sigma_1 t) - M_1 \sin(\sigma_1 t))]}{(C^2 + D^2)} \right] \\
& + \frac{2\mu a_1 h \pi^2}{R_e \alpha^3 \cos^3(\beta s)} \sum_{r=0}^{\infty} (-1)^r r^2 \\
& \times \left[\frac{e^{x_1 t} (x_1 + i\sigma_1)(h^2 + x_1 U\tau)^2}{(x_1^2 + \sigma_1^2)[(h^2 + x_1 U\tau)^2 + lh^4]} + \frac{e^{x_2 t} (x_2 + i\sigma_2)(h^2 + x_2 U\tau)^2}{(x_2^2 + \sigma_2^2)[(h^2 + x_2 U\tau)^2 + lh^4]} \right] \\
& - \left[\frac{a_2 \mu [(M_3 \cos(\sigma_2 t) + M_4 \sin(\sigma_2 t)) + i(M_4 \cos(\sigma_2 t) - M_3 \sin(\sigma_2 t))]}{(G^2 + H^2)} \right] \\
& + \frac{2\mu a_2 h \pi^2}{R_e \alpha^3 \cos^3(\beta s)} \sum_{r=0}^{\infty} (-1)^r r^2 \\
& \times \left[\frac{e^{x_1 t} (x_1 - i\sigma_2)(h^2 + x_1 U\tau)^2}{(x_1^2 + \sigma_2^2)[(h^2 + x_1 U\tau)^2 + lh^4]} + \frac{e^{x_2 t} (x_2 - i\sigma_2)(h^2 + x_2 U\tau)^2}{(x_2^2 + \sigma_2^2)[(h^2 + x_2 U\tau)^2 + lh^4]} \right]
\end{aligned}$$

where

$$\begin{aligned}
b_1 = & R_e U\tau \alpha^2 \cos^2(\beta s), \quad b_2 = R_e h^2(1+l) + U\tau(h^3 C_r + M \operatorname{Re}) \\
& + \alpha^2 \cos^2(\beta s) + r^2 \pi^2 h U\tau,
\end{aligned}$$

$$b_3 = h^2(h^3 C_r + M \operatorname{Re}) \alpha^2 \cos^2(\beta s) + r^2 \pi^2 h,$$

$$x_1 = \frac{-b_2 + \sqrt{b_2^2 - 4b_1 b_3}}{2b_1}, \quad x_2 = \frac{-b_2 - \sqrt{b_2^2 - 4b_1 b_3}}{2b_1},$$

$$R_1 = h^2 C_r + \frac{M \operatorname{Re}}{h} + \frac{R_e \sigma_1^2 l h^3 U\tau}{h(h^4 + \sigma_1^2 U^2 \tau^2)}, \quad R_2 = \frac{R_e \sigma_1^2 l h^3 U\tau}{h(h^4 + \sigma_1^2 U^2 \tau^2)} + \frac{\sigma_1 R_e}{h},$$

$$\alpha_1 = \sqrt{\frac{R_1 + \sqrt{R_1^2 + R_2^2}}{2}}, \quad \beta_1 = \sqrt{\frac{-R_1 + \sqrt{R_1^2 + R_2^2}}{2}},$$

$$A = \sinh(\lambda_1) \cos(\lambda_2), \quad B = \cosh(\lambda_1) \sin(\lambda_2),$$

$$C = \sinh(\theta_1) \cos(\theta_2), \quad D = \cosh(\theta_1) \sin(\theta_2)$$

$$L_1 = \alpha_1 I_1 - \beta_1 J_1, \quad L_2 = \alpha_1 J_1 + \beta_1 I_1, \quad L_3 = \alpha_2 I_2 - \beta_2 J_2, \quad L_4 = \alpha_2 J_2 + \beta_2 I_2,$$

$$M_1 = \alpha_1 C + \beta_1 D, \quad M_2 = \alpha_1 D - \beta_1 C, \quad M_3 = \alpha_2 G + \beta_2 H, \quad M_4 = \alpha_2 H - \beta_2 G,$$

$$\lambda_1 = \alpha_1 (b - \alpha \cos(\beta s)), \quad \lambda_2 = \beta_1 (b - \alpha \cos(\beta s))$$

$$\theta_1 = \alpha_1 (\alpha \cos(\beta s)), \quad \theta_2 = \beta_1 (\alpha \cos(\beta s)), \quad \theta_3 = \alpha_2 (\alpha \cos(\beta s)), \quad \theta_4 = \beta_2 (\alpha \cos(\beta s))$$

$$R_3 = h^2 C_r + \frac{M \operatorname{Re}}{h} + \frac{\operatorname{Re} \sigma_2^2 l h^3 U\tau}{h(h^4 + \sigma_2^2 U^2 \tau^2)}, \quad R_4 = \frac{\sigma_2 \operatorname{Re} l h^5}{h(h^4 + \sigma_2^2 U^2 \tau^2)} + \frac{\sigma_2 \operatorname{Re}}{h}$$

$$\alpha_2 = \sqrt{\frac{R_3 + \sqrt{R_3^2 + R_4^2}}{2}}, \quad \beta_2 = \sqrt{\frac{-R_3 + \sqrt{R_3^2 + R_4^2}}{2}}$$

$$E = \sinh(\lambda_3) \cos(\lambda_4), \quad F = \cosh(\lambda_3) \sin(\lambda_4)$$

$$\begin{aligned}
 G &= \sinh(\theta_3)\cos(\theta_4), \quad F = \cosh(\theta_3)\sin(\theta_4) \\
 \lambda_3 &= \alpha_2(b - \alpha\cos(\beta s)), \quad \lambda_4 = \beta_2(b - \alpha\cos(\beta s)) \\
 \Psi_1 &= AC + BD, \quad \Psi_2 = AD - BC, \quad \Phi_1 = EG + FH, \quad \Phi_2 = EH - FG \\
 \Psi_3 &= h^2\Psi_1 - \sigma_1 U\tau\Psi_2, \quad \Psi_4 = h^2\Psi_2 + \sigma_1 U\tau\Psi_1, \\
 \Phi_3 &= h^2\Phi_1 - \sigma_2 U\tau\Phi_2, \quad \Phi_4 = h^2\Phi_2 + \sigma_2 U\tau\Phi_1 \\
 I_1 &= \cosh(\theta_1)\sinh(\theta_1), \quad J_1 = \sin(\theta_2)\cos(\theta_2), \quad I_2 = \sinh(\theta_3)\cosh(\theta_3), \quad J_2 = \sin(\theta_4)\cos(\theta_4)
 \end{aligned}$$

7. CONCLUSIONS

We can observe the parabolic in nature of velocity profiles for the fluid and dust particles plotted as in Figs. 3 - 5. It is observed that velocity of fluid particles is parallel to velocity of dust particles. Also it is evident from the graphs that, as we increase the strength of the magnetic field, it has an appreciable effect on the velocities of fluid and dust particles. Also, the velocities for fluid and dust particles decrease and reach zero for large values of t which is desirable in physical situation. Fluid and the dust particle velocity are minimum along the centre of the channel. The fluid and dust particle velocity is significantly decreased by the application of the pressure gradient.

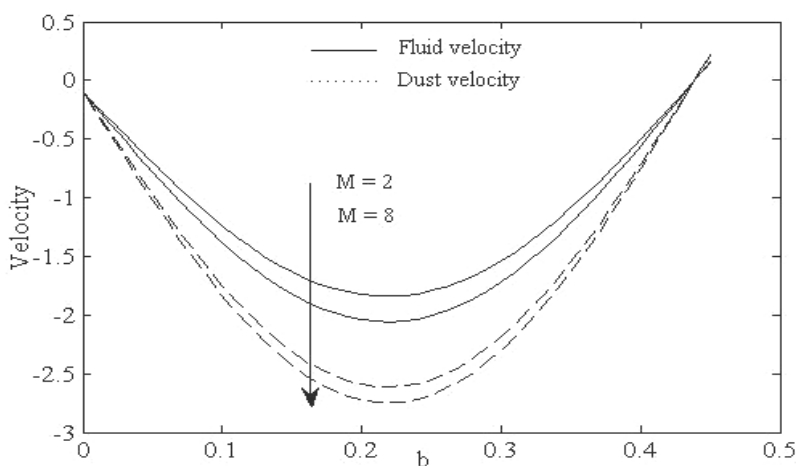


Fig. 3. Variation of both fluid and dust velocity with b for $a_1 = a_2, w_1 = w_2 = w$.

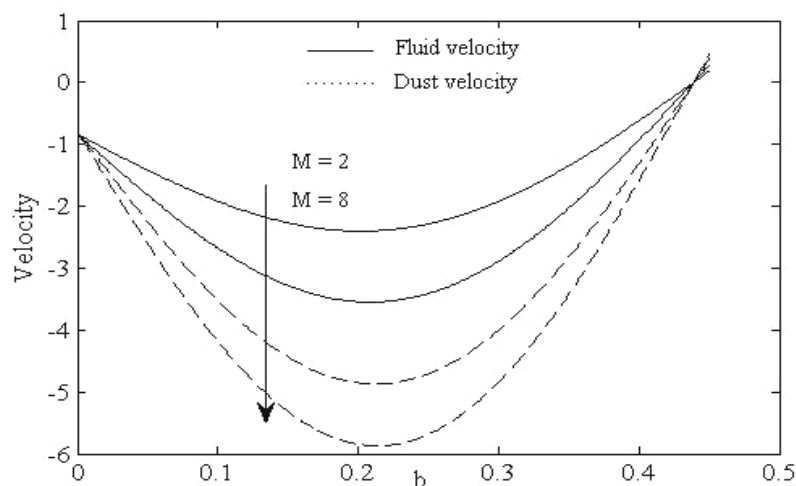


Fig. 4. Variation of both fluid and dust velocity with b for $a_1 = a_2, w_1 = w_2 = w$.



Fig. 5. Variation of both fluid and dust velocity with b for $a_1 = -iu_0$, $a_2 = -iu_0$, $w_1 = w_2 = w$.

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