

ON THE UNIFICATION OF A CLASS OF TRILATERAL GENERATING RELATIONS WITH TCHEBYCHEFF POLYNOMIALS FOR CERTAIN SPECIAL FUNCTION

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Abstract. *In this article we have presented a novel result in connection with the unification of a class of trilateral generating relations for certain special functions with Tchebycheff polynomial by group theoretic method. A good number of applications of our result are also given in section 3 of this paper.*

Keywords: *Trilateral generating relation, special functions.*

AMS-2000 Subject Classification Code: *33C 47.*

1. INTRODUCTION

Generating functions play a large role in the study of special functions. The generating functions for various special functions which are available in the literature are almost bilateral in nature. In recent works, some mixed trilateral generating functions for certain special functions have been investigated by some researchers.

Theories in connection with the unification of bilateral or trilateral generating relations for various special functions are of greater importance in the study of special functions. For previous works in this direction, one can see the works [1-7] and [8-15] in connection with the unification of bilateral and mixed trilateral generating relations.

In this paper, we have made an attempt to present a novel result in connection with the unification of trilateral generating relations for certain special functions by group theoretic method, of course when suitable continuous transformation groups can be constructed for the special function under consideration, with Tchebycheff polynomials. In fact, this method [16] is based on the theory of one parameter group of continuous transformations by means of which any unilateral generating relation involving a special function can be transformed in to a bilateral generating relation and then into a trilateral generating relation with Tchebycheff polynomial by means of the relation [17]:

$$(1.1) \quad T_n(x) = \frac{1}{2} \left[\left(x + \sqrt{x^2 - 1} \right)^n + \left(x - \sqrt{x^2 - 1} \right)^n \right].$$

The detailed discussion is given below:

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2. GROUP-THEORETIC DISCUSSION

Let us first consider the following unilateral generating relation,

$$(2.1) \quad G(x, w) = \sum_{n=0}^{\infty} a_n p_n^{(\alpha)}(x) w^n,$$

where $p_n^{(\alpha)}(x)$ is a special function of degree n and of parameter α and a_n is independent of x, w .

Replacing w by vwz and then multiplying both sides of (2.1) by y^α we get,

$$(2.2) \quad y^\alpha G(x, vwz) = \sum_{n=0}^{\infty} a_n \left(p_n^{(\alpha)}(x) y^\alpha z^n \right) (vw)^n.$$

Let us suppose that for the above special function, it is possible to define a linear partial differential operator R , which generates a continuous transformation groups as follows:

$$R = \xi(x, y, z) \frac{\partial}{\partial x} + \eta(x, y, z) \frac{\partial}{\partial y} + \zeta(x, y, z) \frac{\partial}{\partial z} + \theta(x, y, z)$$

such that

$$(2.3) \quad R \left(p_n^{(\alpha)}(x) y^\alpha z^n \right) = \rho_n p_{n+1}^{(\alpha-1)}(x) y^{\alpha-1} z^{n+1}$$

and

$$(2.4) \quad e^{wR} f(x, y, z) = \Omega(x, y, z, w) f(g(x, y, z, w), h(x, y, z, w), k(x, y, z, w)).$$

Operating both sides of (2.2) by e^{wR} , we get

$$(2.5) \quad e^{wR} \left(y^\alpha G(x, vwz) \right) = e^{wR} \left(\sum_{n=0}^{\infty} a_n \left(p_n^{(\alpha)}(x) y^\alpha z^n \right) (vw)^n \right).$$

The left number of (2.5), with the help of (2.4), becomes

$$(2.6) \quad \Omega(x, y, z, w) \left(h(x, y, z, w) \right)^\alpha G \left(g(x, y, z, w), vwk(x, y, z, w) \right).$$

The right number of (2.5), with the help of (2.3), becomes

$$(2.7) \quad \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} a_n \frac{w^k}{k!} \rho_n \rho_{n+1} \dots \rho_{n+k-1} p_{n+k}^{(\alpha-k)}(x) y^{\alpha-k} z^{n+k} (vw)^n.$$

Now equating (2.6) and (2.7) and then putting $y = z = 1$, we get

$$(2.8) \quad \Omega(x, 1, 1, w) \left(h(x, 1, 1, w) \right)^\alpha G \left(g(x, 1, 1, w), vwk(x, 1, 1, w) \right)$$

$$\begin{aligned}
 &= \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} a_n \frac{w^{n+k}}{k!} \rho_n \rho_{n+1} \dots \rho_{n+k-1} p_n^{(\alpha-k)}(x) v^n \\
 &= \sum_{n=0}^{\infty} \sum_{k=0}^n a_{n-k} \frac{w^n}{k!} \rho_{n-k} \rho_{n-k+1} \dots \rho_{n-1} p_n^{(\alpha-k)}(x) v^{n-k} \\
 &= \sum_{n=0}^{\infty} w^n \sum_{k=0}^n a_k \frac{\rho_k \rho_{k+1} \dots \rho_{n-1}}{(n-k)!} p_n^{(\alpha-n+k)}(x) v^k \\
 &= \sum_{n=0}^{\infty} w^n \sum_{k=0}^n a_k \frac{\prod_{i=k}^{n-1} \rho_i}{(n-k)!} p_n^{(\alpha-n+k)}(x) v^k \\
 &= \sum_{n=0}^{\infty} w^n \sigma_n(x, v),
 \end{aligned}$$

where

$$(2.9) \quad \sigma_n(x, v) = \sum_{k=0}^n a_k \frac{\prod_{i=k}^{n-1} \rho_i}{(n-k)!} p_n^{(\alpha-n+k)}(x) v^k .$$

Now to convert the above bilateral generating relation into a trilateral generating relation with Tchebycheff polynomial, we notice that

$$\begin{aligned}
 &\sum_{n=0}^{\infty} \sigma_n(x, v) T_n(u) w^n \\
 &= \sum_{n=0}^{\infty} \sigma_n(x, v) \frac{1}{2} \left[(u + \sqrt{u^2 - 1})^n + (u - \sqrt{u^2 - 1})^n \right] w^n \\
 &= \frac{1}{2} \left[\sum_{n=0}^{\infty} \sigma_n(x, v) \left\{ w (u + \sqrt{u^2 - 1}) \right\}^n + \sum_{n=0}^{\infty} \sigma_n(x, v) \left\{ w (u - \sqrt{u^2 - 1}) \right\}^n \right] \\
 &= \frac{1}{2} \left[\sum_{n=0}^{\infty} \sigma_n(x, v) \rho_1^n + \sum_{n=0}^{\infty} \sigma_n(x, v) \rho_2^n \right] \\
 &= \frac{1}{2} \left[\Omega(x, 1, 1, \rho_1) (h(x, 1, 1, \rho_1))^\alpha G(g(x, 1, 1, \rho_1), v \rho_1 k(x, 1, 1, \rho_1)) \right. \\
 &\quad \left. + \Omega(x, 1, 1, \rho_2) (h(x, 1, 1, \rho_2))^\alpha G(g(x, 1, 1, \rho_2), v \rho_2 k(x, 1, 1, \rho_2)) \right],
 \end{aligned}$$

where

$$\rho_1 = w (u + \sqrt{u^2 - 1}) \quad \text{and} \quad \rho_2 = w (u - \sqrt{u^2 - 1}) .$$

Thus we have prove the following theorem.

Theorem 1. If there exists a unilateral generating relation of the form:

$$G(x, w) = \sum_{n=0}^{\infty} a_n p_n^{(\alpha)}(x) w^n$$

then

$$\sum_{n=0}^{\infty} \sigma_n(x, v) T_n(u) w^n$$

$$= \frac{1}{2} \left[\Omega(x, 1, 1, \rho_1) (h(x, 1, 1, \rho_1))^\alpha G(g(x, 1, 1, \rho_1), v\rho_1 k(x, 1, 1, \rho_1)) \right. \\ \left. + \Omega(x, 1, 1, \rho_2) (h(x, 1, 1, \rho_2))^\alpha G(g(x, 1, 1, \rho_2), v\rho_2 k(x, 1, 1, \rho_2)) \right],$$

where

$$\sigma_n(x, v) = \sum_{k=0}^n a_k \frac{\prod_{i=k}^{n-1} \rho_i}{(n-k)!} p_n^{(\alpha-n+k)}(x) v^k, \\ \rho_1 = w(u + \sqrt{u^2 - 1}) \quad \text{and} \quad \rho_2 = w(u - \sqrt{u^2 - 1}).$$

The above theorem does not seem to have appeared in the earlier works. We now proceed to give a good number of applications of our result.

3. APPLICATIONS

Application 1. At first we take

$$(3.1) \quad p_n^{(\alpha)}(x) = L_n^{(\alpha)}(x).$$

Now we consider the operator R [18], where

$$R = xy^{-1}z \frac{\partial}{\partial x} + z \frac{\partial}{\partial y} - xy^{-1}z$$

such that

$$(3.2) \quad R \left(L_n^{(\alpha)}(x) y^\alpha z^n \right) = (n+1) L_{n+1}^{(\alpha-1)}(x) y^{\alpha-1} z^{n+1}$$

and

$$(3.3) \quad e^{wR} f(x, y, z) = \exp(-wxy^{-1}z) f \left(x(1 + wy^{-1}z), y(1 + wy^{-1}z), z \right).$$

So by comparing (3.2), (3.3) with (2.3), (2.4), we get

$$\rho_n = (n+1), \quad \Omega(x, y, z, w) = \exp(-wxy^{-1}z), \\ g(x, y, z, w) = x(1 + wy^{-1}z), \quad h(x, y, z, w) = y(1 + wy^{-1}z), \\ k(x, y, z, w) = z.$$

Then, by the application of our theorem, we get the following result on trilateral generating relations with Tchebycheff polynomials.

Theorem 2. If

$$(3.4) \quad G(x, w) = \sum_{n=0}^{\infty} a_n L_n^{(\alpha)}(x) w^n$$

then

$$(3.5) \quad \sum_{n=0}^{\infty} \sigma_n(x, v) T_n(u) w^n \\ = \frac{1}{2} \left[\exp(-\rho_1 x) (1 + \rho_1)^\alpha G(x(1 + \rho_1), v\rho_1) + \exp(-\rho_2 x) (1 + \rho_2)^\alpha G(x(1 + \rho_2), v\rho_2) \right],$$

where

$$\sigma_n(x, v) = \sum_{k=0}^n a_k \binom{n}{k} L_n^{(\alpha-n+k)}(x) v^k,$$

$$\rho_1 = w(u + \sqrt{u^2 - 1}) \quad \text{and} \quad \rho_2 = w(u - \sqrt{u^2 - 1}),$$

which dose not seem to appear before.

Application 2. Now we take

$$(3.6) \quad p_n^{(\alpha)}(x) = f_n^\beta(x) \quad \text{with} \quad \alpha = \beta,$$

and we consider the following partial differential operator R [19] :

$$R = xy^{-1}z \frac{\partial}{\partial x} - z \frac{\partial}{\partial y} - (x-1)y^{-1}z$$

such that

$$(3.7) \quad R \left(f_n^\beta(x) y^\beta z^n \right) = -(n+1) f_{n+1}^{\beta-1}(x) y^{\beta-1} z^{n+1}$$

and

$$(3.8) \quad e^{wR} f(x, y, z) = \left(\frac{y}{y-wz} \right) \exp\left(\frac{-xwz}{y-zw} \right) f\left(\frac{xy}{y-zw}, y-zw, z \right).$$

Then by comparing (3.7), (3.8) with (2.3), (2.4), we get

$$\rho_n = -(n+1), \quad \Omega(x, y, z, w) = \left(\frac{y}{y-wz} \right) \exp\left(\frac{-xwz}{y-zw} \right),$$

$$g(x, y, z, w) = \left(\frac{xy}{y-wz} \right), \quad h(x, y, z, w) = y - wz, \quad k(x, y, z) = z.$$

Therefore, by the application of our Theorem 1, we get the following result on trilateral generating relation with Tchebycheff polynomials.

Theorem 3. If

$$(3.9) \quad G(x, w) = \sum_{n=0}^{\infty} a_n f_n^\beta(x) w^n$$

then

$$(3.10) \quad \sum_{n=0}^{\infty} \sigma_n(x, v) T_n(u) w^n = \frac{1}{2} \left[(1 + \rho_1)^{\beta-1} \exp\left(\frac{x\rho_1}{1 + \rho_1} \right) G\left(\frac{x}{1 + \rho_1}, v\rho_1 \right) + (1 + \rho_2)^{\beta-1} \exp\left(\frac{x\rho_2}{1 + \rho_2} \right) G\left(\frac{x}{1 + \rho_2}, v\rho_2 \right) \right]$$

where

$$\sigma_n(x, v) = \sum_{k=0}^n a_n \binom{n}{k} f_n^{(\beta-n+k)}(x) v^k,$$

$$\rho_1 = w(u + \sqrt{u^2 - 1}) \quad \text{and} \quad \rho_2 = w(u - \sqrt{u^2 - 1}),$$

which does not seem to appear before.

Application 3. We now take

$$p_n^{(\alpha)}(x) = Y_n^{(\alpha)}(x)$$

Then from [20], we notice that

$$R = x^2 y^{-1} z \frac{\partial}{\partial x} + xz \frac{\partial}{\partial y} + x y^{-1} z^2 \frac{\partial}{\partial z} + (\beta - x)y^{-1}z$$

such that

$$(3.11) \quad R \left(Y_n^{(\alpha)}(x) y^\alpha z^n \right) = \beta Y_{n+1}^{(\alpha-1)}(x) y^{\alpha-1} z^{n+1}$$

and

$$(3.12) \quad e^{wR} f(x, y, z) = (1 - wxy^{-1}z) \exp(\beta wy^{-1}z) \times f\left(\frac{x}{1 - wxy^{-1}z}, \frac{y}{1 - wxy^{-1}z}, \frac{z}{1 - wxy^{-1}z}\right).$$

Comparing (3.11), (3.12) with (2.3), (2.4), we get

$$\rho_n = \beta, \quad \Omega(x, y, z, w) = (1 - wxy^{-1}z) \exp(\beta wy^{-1}z),$$

$$g(x, y, z, w) = \frac{x}{1 - wxy^{-1}z}, \quad h(x, y, z, w) = \frac{y}{1 - wxy^{-1}z},$$

$$k(x, y, z, w) = \frac{z}{1 - wxy^{-1}z}.$$

Then by the application of our theorem, we get the following result on trilateral generating relation with Tchebycheff polynomials.

Theorem 4. If

$$(3.13) \quad G(x, w) = \sum_{n=0}^{\infty} a_n Y_n^{(\alpha)}(x) w^n$$

then

$$(3.14) \quad \sum_{n=0}^{\infty} \sigma_n(x, v) T_n(u) w^n = \frac{1}{2} \left[\exp(\beta \rho_1) (1 - \rho_1 x)^{1-\alpha} G\left(\frac{x}{1 - \rho_1 x}, \frac{v \rho_1}{1 - \rho_1 x}\right) + \exp(\beta \rho_2) (1 - \rho_2 x)^{1-\alpha} G\left(\frac{x}{1 - \rho_2 x}, \frac{v \rho_2}{1 - \rho_2 x}\right) \right]$$

where

$$(3.15) \quad \sigma_n(x, v) = \sum_{k=0}^n a_k \frac{\beta^{n-k}}{(n-k)!} Y_n^{(\alpha-n+k)}(x) v^k,$$

$$\rho_1 = w(u + \sqrt{u^2 - 1}) \quad \text{and} \quad \rho_2 = w(u - \sqrt{u^2 - 1}),$$

which does not seem to be appear before.

Application 4. We now take

$$p_n^{(\alpha)}(x) = C_n^\lambda(x) \quad \text{with } \alpha = \lambda .$$

Then from [21], we notice that

$$R = (x^2 - 1)y^{-1}z \frac{\partial}{\partial x} + 2x z \frac{\partial}{\partial y} - xy^{-1}z$$

such that

$$(3.16) \quad R \left(C_n^\lambda(x)y^\lambda z^n \right) = \frac{(n + 2\lambda - 1)(n + 1)}{2(\lambda - 1)} C_{n+1}^{\lambda-1}(x)y^{\lambda-1}z^{n+1}$$

and

$$(3.17) \quad e^{wR} f(x, y, z) = \left\{ 1 + 2wxy^{-1}z + (x^2 - 1)w^2y^{-2}z^2 \right\}^{-1/2} \\ \times f \left(x + w(x^2 - 1)y^{-1}z, y \left\{ 1 + 2wxy^{-1}z + (x^2 - 1)w^2y^{-2}z^2 \right\}, z \right).$$

Now comparing (3.16), (3.17) with (2.3), (2.4) , we get

$$\rho_n = \frac{(n + 2\lambda - 1)(n + 1)}{2(\lambda - 1)}, \quad \Omega(x, y, z, w) = \left\{ 1 + 2wxy^{-1}z + (x^2 - 1)w^2y^{-2}z^2 \right\}^{-1/2}$$

$$g(x, y, z, w) = x + w(x^2 - 1)y^{-1}z, \quad h(x, y, z, w) = y \left\{ 1 + 2wxy^{-1}z + (x^2 - 1)w^2y^{-2}z^2 \right\}, \\ k(x, y, z, w) = z$$

Then by the application of our theorem, we get on simplification the following result on trilateral generating relation with Tchebycheff polynomials.

Theorem 5. If

$$(3.18) \quad G(x, w) = \sum_{n=0}^{\infty} a_n C_n^\lambda(x) w^n$$

then

$$(3.19) \quad \sum_{n=0}^{\infty} \sigma_n(x, v) T_n(u) w^n \\ = \frac{1}{2} \left[\left\{ 1 + 4\rho_1x + 4\rho_1^2(x^2 - 1) \right\}^{\lambda-1/2} G \left(x + 2\rho_1(x^2 - 1), \rho_1v \right) \right. \\ \left. + \left\{ 1 + 4\rho_2x + 4\rho_2^2(x^2 - 1) \right\}^{\lambda-1/2} G \left(x + 2\rho_2(x^2 - 1), \rho_2v \right) \right],$$

where

$$\sigma_n(x, v) = \sum_{k=0}^n a_k \binom{n}{k} \frac{(-2\lambda - k + 1)_{n-k}}{(-\lambda + 1)_{n-k}} C_n^{\lambda-n+k}(x) v^k ,$$

$$\rho_1 = w(u + \sqrt{u^2 - 1}) \quad \text{and} \quad \rho_2 = w(u - \sqrt{u^2 - 1}),$$

which does not seem to have appeared in the earlier works.

Application 5. We now take

$$p_n^{(\alpha)}(x) = P_n^{(\alpha, \beta)}(x) .$$

Then we consider the operator R [22, 23], where

$$R = (1-x^2)y^{-1}z \frac{\partial}{\partial x} + (1-x)z \frac{\partial}{\partial y} - (1+x)y^{-1}z^2 \frac{\partial}{\partial z} - (1+\alpha)(1+x)y^{-1}z.$$

such that

$$(3.20) \quad R \left(P_n^{(\alpha, \beta)}(x) y^\beta z^n \right) = -2(n+1) P_{n+1}^{(\alpha, \beta-1)}(x) y^{\beta-1} z^{n+1}$$

and

$$(3.21) \quad e^{wR} f(x, y, z) = \left\{ 1 + w(1+x)y^{-1}z \right\}^{-1-\alpha} \\ \times f \left(\frac{x + w(1+x)y^{-1}z}{1 + w(1+x)y^{-1}z}, \frac{y(1+2wy^{-1}z)}{1 + w(1+x)y^{-1}z}, \frac{z}{1 + w(1+x)y^{-1}z} \right).$$

Comparing (3.20), (3.21) with (2.3), (2.4), we get

$$\rho_n = -2(n+1), \quad \Omega(x, y, z, w) = \{1 + w(1+x)y^{-1}z\}^{-1-\alpha}, \\ g(x, y, z, w) = \frac{x + w(1+x)y^{-1}z}{1 + w(1+x)y^{-1}z}, \quad h(x, y, z, w) = \frac{y(1+2wy^{-1}z)}{1 + w(1+x)y^{-1}z}, \\ k(x, y, z, w) = \frac{z}{1 + w(1+x)y^{-1}z}.$$

Then by the application of our theorem, we get on simplification the following result on trilateral generating relation with Tchebycheff polynomials.

Theorem 6. If

$$(3.22) \quad G(x, w) = \sum_{n=0}^{\infty} a_n P_n^{(\alpha, \beta)}(x) w^n,$$

then

$$(3.23) \quad \sum_{n=0}^{\infty} \sigma_n(x, v) T_n(u) w^n \\ = \frac{1}{2} \left[\left\{ 1 - (1+x) \frac{\rho_1}{2} \right\}^{-1-\alpha-\beta} (1-\rho_1)^\beta G \left(\frac{x - (1+x) \frac{\rho_1}{2}}{1 - (1+x) \frac{\rho_1}{2}}, \frac{\rho_1 v}{1 - (1+x) \frac{\rho_1}{2}} \right) \right. \\ \left. + \left\{ 1 - (1+x) \frac{\rho_2}{2} \right\}^{-1-\alpha-\beta} (1-\rho_2)^\beta G \left(\frac{x - (1+x) \frac{\rho_2}{2}}{1 - (1+x) \frac{\rho_2}{2}}, \frac{\rho_2 v}{1 - (1+x) \frac{\rho_2}{2}} \right) \right]$$

where

$$\sigma_n(x, v) = \sum_{k=0}^n a_k \binom{n}{k} P_n^{(\alpha, \beta-n+k)}(x) v^k,$$

$$\rho_1 = w(u + \sqrt{u^2 - 1}) \quad \text{and} \quad \rho_2 = w(u - \sqrt{u^2 - 1}).$$

Now if in place of R , we consider the following operator R_1 [24]

$$R_1 = (1-x^2)y^{-1}z \frac{\partial}{\partial x} - (1-x)z \frac{\partial}{\partial y} + (1-x)y^{-1}z^2 \frac{\partial}{\partial z} + (1-x)y^{-1}z(1+\beta)$$

such that

$$(3.24) \quad R_1 \left(P_n^{(\alpha, \beta)}(x) y^\alpha z^n \right) = -2(n+1) P_{n+1}^{(\alpha-1, \beta)}(x) y^{\alpha-1} z^{n+1}$$

and

$$(3.25) \quad e^{wR_1} f(x, y, z) = \{ 1 + w(x-1)y^{-1}z \}^{-1-\beta} \times f \left(\frac{x - w(x-1)y^{-1}z}{1 + w(x-1)y^{-1}z}, \frac{y(1 - 2wy^{-1}z)}{1 + w(x-1)y^{-1}z}, \frac{z}{1 + w(x-1)y^{-1}z} \right).$$

Then by the application of our theorem, we get the following result (analogous to Theorem 6) on bilateral generating relation involving Jacobi polynomial.

Theorem 7. If

$$(3.26) \quad G(x, w) = \sum_{n=0}^{\infty} a_n P_n^{(\alpha, \beta)}(x) w^n$$

then

$$(3.27) \quad \sum_{n=0}^{\infty} \sigma_n(x, v) T_n(u) w^n = \frac{1}{2} \left[\{ 1 - (x-1)\frac{\rho_1}{2} \}^{-1-\alpha-\beta} (1 + \rho_1)^\alpha G \left(\frac{x + (x-1)\frac{\rho_1}{2}}{1 - (x-1)\frac{\rho_1}{2}}, \frac{\rho_1 v}{1 - (x-1)\frac{\rho_1}{2}} \right) + \{ 1 - (x-1)\frac{\rho_2}{2} \}^{-1-\alpha-\beta} (1 + \rho_2)^\alpha G \left(\frac{x + (x-1)\frac{\rho_2}{2}}{1 - (x-1)\frac{\rho_2}{2}}, \frac{\rho_2 v}{1 - (x-1)\frac{\rho_2}{2}} \right) \right]$$

where

$$\sigma_n(x, v) = \sum_{k=0}^n a_k \binom{n}{k} P_n^{(\alpha-n+k, \beta)}(x) v^k .$$

It may be pointed out that the Theorem 7 can be directly obtained from Theorem 6 by using the symmetry relation [17]

$$P_n^{(\alpha, \beta)}(x) = (-1)^n P_n^{(\beta, \alpha)}(x).$$

4. CONCLUSION

From the above discussion, it is clear that one may apply Theorem-1 in the case of other polynomials and functions existing in the field of special functions subject to the condition of construction of continuous transformation groups for the said special function. Furthermore, the importance of the above theorems (2-7) lies in the fact that whenever one knows a unilateral generating relation of the form (3.4, 3.9 etc) then the corresponding trilateral generating functions can at once be written down from (3.5, 3.10 etc). Thus, one can get a large number of trilateral generating functions with Tchebycheff polynomials by attributing different suitable values to a_n .

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