ORIGINAL PAPER

AN INTERESTING DIOPHANTINE EQUATION

MARINA TOMA[1]

Manuscript received: 02.11.2011; Accepted paper: 21.11.2011;

Published online: 01.12.2011.

Abstract*. In this article it is solved an interesting diophantine equation in a very ingenious way.*

Keywords*: diophantine equation, the coconuts problem.*

1. INTRODUCTION

Diophantus from Alexandria was a famous greek mathematician who lived around 325-403 a.Ch. and it is considered to be „the father of algebra”. He developed mathematical formulas for the calculation of equations and he wrote a texbook on arithmetic. From the 13 books of his main work “Arithmetica” only 6 survived. Before Diophantus all algebra was expressed without any symbolism.

Diophantus` Arithmetica arrived in Europe with the help of the arabian mathematicians. Diophantus wrote about a special kind of equations named as diophantic equations which he described in numerous forms wihout indicating a general method for obtaining a solution.

A legend is saying that on his grave there is a stone saying how many years he had lived. The text says like that: here is buried Diophantus, the sitxth part of his life was his beautiful childhood, the 12/th part of his life was his youth. After another 7/th part of his life he got married and 5 years later he got a son but his son lived only half of his father life. Remaing in great suffering the old man died 4 years later than his son. We can calculate that he was 84 years old when he died.

2. CONTENT

In Mathematics a Diophantine equation is an indeterminate polynomial equation that allows the variables to be only integers. We can see some examples of this kind of equations:

$ax + by = 1$ a linear Diophantine equation

$x^n + y^n = z^n$ For n=2 there are an infinit number of solution respectively any group of numbers (x, y, z) called Pythagorean triples.

For larger values of n Fermat`s last theorem states that no positive integer solution (x, y, z) exist. This theorem remained unsolved for more than 350 years.

[1] Valahia University of Targoviste, Faculty of Science and Arts, 130024, Targoviste, Romania.
E-mail: tmmarina@yahoo.com.

$x^2 - ny^2 = \pm 1$ these are named Pell`s equations because of the English mathematician John Pell. They were studied by Brahmagupta in the 7/th century as well as by Fermat in the 17/th century.

$\frac{4}{n} = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ The Erdös-Straus conjecture states that for every positive integer $n \geq 0$ there exists a solution with positive integers.

Regarding any Diophantine equation we have to answer to the following questions:

Are there any solutions?

The number of solutions are finit or infinit?

Can all solutions be found?

An example of an infinite Diophantine equation can be expressed like this: How many ways can a given integer N be written as the sum of a square plus twice a square plus thrice a square and so on?

Regarding a special linear diophantine equation there is a very nice story. The Saturday Evening Post from America published on the date of 9 of October 1926 a short story named The Coconuts written by Ben Ames Willimas. The story told about a contractor who was also pasionate for mathematics. He had a rival who, in order to keep him busy for not involving in a project which he wanted for himself, send him the following problem: five men and a monkey had wrecked on a deserted island. They all gathered a large number of coconuts in the firs day of their staying there. Being too tired they decided to sleep first and to dived the coconuts early in the next morning. During the night one of the men woke up and decided to take his part. So he put aside one coconut for the monkey, the rest of the coconuts he devided in five parts, took one part and hide it and after that put the remained coconuts back. After a while the second man woke up and did the same and so on, all the men did that one by one. In the morning they all woke up, devide the coconuts they find in 5 equal parts and took one part everyone of them. The question was how many coconuts they gathered in their first day on the island? The story said the the contractor was so determined to solve this problem that he forgot about the project he wanted and so his rival had no more problem in getting it for himself.

When it was published this problem it was no solution indicated so the paper`s office was invaded by a huge number of letters and telegrams asking for the right answer and this went on for more than 20 years. Maybe it was the diophantine equation with the smallest number of solutions in the spite of the large amount of time envolved.

In fact the problem of the coconuts was not new. It was an older version in which in the morning also one coconut is taken away for the monkey. Both this one and the one modificated by Williams have an infinite number of solutions and we just have to find the smallest of them.

It can be obtained a solution in the classic way and this is the number 3121.

There exist another solution extremely beautiful, some are saying that this was discovered by Professor Dirac from Cambridge University, others saying that it belongs to Proffesor Whitehead from Oxford University. This solution uses the notion of negative number of coconuts.

We can consider that N is the number we want to find. Since N is devided six times in five parts it is obvious that 5^6 sau 15625 can be added to any solution and we will get another solution.

Next we can notice that -4 is a solution. Indeed after takeing one coconut for the monkey we will have -5 coconuts, we will devide them in five equal parts every part of -1 coconut, every man will take one part out and so we will have the remaining -4 coconuts again. After that all we have to do is to add 15625 to that negative solution and so we will find the real solution to that problem which was indeed 15621.

This process can also be generalized: if we have 4 sailors, we can start with 3 negative coconuts and then add 4^5, if we have 6 sailors we start with 5 negative coconuts and then add 6^7 and so on.

3. CONCLUSION

In regards of the Williams version of the problem we allready know that $5^5 - 4$ which is 3121 is the smallest number which can be devided five times and the monkey will get one coconut each time. After those five operations they will remain 1020 coconuts and we can easily see that this number can be devided with 5 so it is possibil the sixth process and the monkey this time will get nothing.

REFERENCES

[1] Gardner, M., *Amuzamente Matematice,* Ed. Ştiinţifică, Bucureşti, 1968.
[2] Purcaru, I., Bască, O., *Oameni, idei, fapte din istoria matematicii*, Ed. Economică, 1996.