ORIGINAL PAPER

A NOTE ON TOTAL GLOBAL DOMINATION OF A GRAPH

YANAMANDRAM B. VENKATAKRISHNAN[1], VENKATA SWAMINATHAN[2]

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Abstract. *Bipartite theory of graphs was formulated by Stephen Hedetniemi and Renu Laskar in which concepts in graph theory have equivalent formulations as concepts for bipartite graphs. We give the bipartite version of total global dominating sets.*

Keywords: *Bipartite theory, Total global dominating set, Signed Y −dominating set.*

1. INTRODUCTION

The graphs G considered here have order p and size q and both G and their complements $\overline{G}$ have no isolates. A set D of vertices in a graph $G=(V,E)$ is a dominating set of G if every vertex in $V-D$ is adjacent to some vertex in D. The domination number $\gamma(G)$ of G is the minimum cardinality of a total global domination set.

A dominating set D of G is a total dominating set[4], if for every $u \in V(G)$ there exists $v \in D$ such that u and v are adjacent. The total domination $\gamma_t(G)$ of G of G is the minimum cardinality of a total dominating set.

A total dominating set T of a graph G is a total global dominating set [4] if T is also a total dominating set of $\overline{G}$.The total global domination number $\gamma_{tg}(G)$ is the minimum cardinality of a total global dominating set.Variety of domination has been studied extensively by researchers and for a brief introduction to the theory of domination in graphs, the reader is directed to the books [1, 2].

1.1. BIPARTITE CONSTRUCTIONS

Given an arbitrary graph G, we can construct variety of bipartite graphs $G^1=(X,Y,E^1)$ which faithfully represents G, in the sense that given two graphs G and H, G is isomorphic to H if and only if the corresponding bipartite graphs G^1 and H^1 are isomorphic.

We give below the bipartite construction super duplicate graph $D^*(G)$ as defined in [3].

Super Duplicate graph $D^*(G)$: The bipartite graph $D^*(G)=(V, V^1,E^{11})$ is defined by the edges $E^{11}=\left\{\left(u,v^1\right):\left(u,v\right)\notin E(G)\right\}\cup\left\{\left(u,v^1\right),\left(u^1,v\right):\left(u,v\right)\in E(G)\right\}$.

[1] Department of Mathematics,School of Humanities and Sciences, SASTRA University,Tanjore, India.
E-mail: venkatakrish2@maths.sastra.edu
[2] Reader(Retd.), Ramanujan Research center, Saraswathi Narayanan College,Madurai, India.
E-mail: sulanesri@yahoo.com

2. BIPARTITE THEORY OF GRAPHS

Bipartite theory of graphs was formulated by Stephen Hedetniemi and Renu Laskar in which concepts in graph theory have equivalent formulations as concepts for bipartite graphs. Equivalently, given any problem, say P, on an arbitrary graph *G*, there is very likely a corresponding problem Q on a bipartite graph G^1, such that a solution for *Q* provides a solution for P. One such reformulation is the concept of Y −dominating set.

Let *G=(X, Y, E)* be a bipartite graph. A subset *D* of *X* is called an Y −dominating set [5] if every $y \in Y$ is adjacent to a vertex of *D*. The minimum cardinality of an Y −dominating set is called the Y –domination number of *G* and is denoted by $\gamma_Y(G)$.

Bipartite theory on domination in complement of a graph, neighbourhood set of a graph and global dominating set of a graph was discussed in [7, 8].We present the bipartite theory of total global dominating set of a graph.

2.1. BIPARTITE THEORY OF TOTAL DOMINATION

We assign symbols to the edges of the super Duplicate graph obtained from an arbitrary graph *G* as follows: The function *f* is defined as: $f : E(D^*(G)) \to \{-,+\}$ such that uv^1 is assigned a + sign if $uv \in E(G)$ and uv^1 is assigned a − sign if $uv \notin E(G)$.

Let *G=(X, Y, E)* be a bipartite graph. A vertex $x \in X$ positively (negatively) dominates $y \in Y$, if there exists an edge $e = xy$ with sign + (− respectively). A subset *D* of *X* is a signed Y − dominating set if for every $y \in Y$ there exists two vertices x_1 and x_2 in *D* such that the edges x_1y and x_2y are of different signs. The signed Y − domination number $\gamma_s y(G)$ of *G* is the minimum cardinality of a signed Y − dominating set.

Theorem: 1 For an arbitrary graph *G = (V, E)* with no isolates in *G* and $\overline{G}, \gamma_s y(D^*(G) = \gamma_{tg}(G)$.

Proof: Let *S* be a γ_{tg} -set of *G*. Then *S* is a total dominating set in both *G* and $\overline{G}$. For every vertex $v \in V$, there exists two vertices u_1 and u_2, $u_1 = u_2$ in *S* adjacent to *v* such that $u_1 v \in E(G)$ and $u_2 v \in E(\overline{G})$. In the graph $D^*(G) = (V, V^1, E^1)$, the vertex $v^1 \in V^1$ is adjacent to u_1 and u_2 in *S* contained in *V*. The edges u_1v^1 is an edge with sign + and u_2v^1 is an edge with sign -. Therefore, *S* is a signed Y − dominating set in $D^*(G)$. Hence,

$$\gamma_s y(D^*(G)) \le |S| = \gamma_{tg}(G).$$ ■

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