ORIGINAL PAPER

COMMUTATIVITY OF SEMI NEAR RINGS

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Abstract. *In this paper we have mainly obtained some results related to commutativity of cancellative semi near rings with identity element.*

Keywords: *Semi near ring, Cancellative semi near ring, identity element.*

AMS subject classification (2000): *16Y60, 16W50.*

1. INTRODUCTION

V. G. Van Hoorn and B. Van Rootselaar [4] discussed general theory of semi near rings. In this paper we provided some necessary and sufficient conditions for the commutativity of cancellative semi near rings with identity element.

2. PRELIMINARIES

Definition 1: A nonempty set N together with two binary operations '+' and '.' is said to be a semi near ring if (N, +) is a semi group and (N, .) is a semi group satisfying the distributive laws.

Definition 2: A semi near ring N is cancellative if the following conditions hold $\forall a, b, c \in N$.

(1) $a+b=a+c \Rightarrow b=c$ (2) $b+a=c+a \Rightarrow b=c$

Definition 3: In a semi near ring N if there exists an element 'e' such that $a.e=e.a=a$ $\forall a \in N$ then N is called a semi near ring with identity element.

We start with the following theorem.

Theorem 1: A cancellative semi near ring N with identity is commutative if and only if $x.y^2 = y.x.y \quad \forall x, y \in N$.

Proof: First part is trivial. For the converse replace theelement y with y + e. We get ,

$$x.(y+e)^2 = (y+e).x.(y+e)$$
$$\Rightarrow x.(y+e).(y+e) = (y.x+x)(y+e)$$
$$\Rightarrow (xy+x).(y+e) = (y.x).y + y.x + x.y + x$$
$$\Rightarrow (x.y).y + x.y + x.y + x = (y.x).y + y.x + x.y + x$$
$$\Rightarrow (x.y).y + x.y = (y.x).y + y.x \quad \text{(By Cancellation law)}$$
$$\Rightarrow x(y.y) + x.y = y.x.y + y.x \quad \text{(By Associativity)}$$
$$\Rightarrow x.y^2 + x.y = y.x.y + y.x \quad \text{(By the given condition)}$$
$$\Rightarrow x.y = y.x \quad \forall x, y \in N$$

Hence N is a commutative semi near ring.

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Lemma 2: A cancellative semi near ring N with identity is commutative if and only if $x^2.y = x.y.x \ \ \forall x, y \in N$.

Proof: Trivial

Theorem 3: A cancellative semi near ring N with identity is commutative if and only if $x.y^n = y.x.y^{n-1}$, $\forall x, y \in N$ and $n \geq 2$.

Proof: This theorem can be proved using principle of Mathematical Induction.

If n=2 then the theorem follows from Theorem (1).

If the result is true for n = k, then N is commutative iff $x.y^k = y.x.y^{k-1} \ \forall x, y \in N$

$$x.y^{k+1} = x.y^k.y = y.x.y^{k-1}.y = y.x.y^k \qquad \forall x, y \in N$$

Hence the Theorem follows from the principle of Mathematical induction.

Lemma 4: A cancellative semi near ring with identity N is commutative if and only if $y^n.x = y^{n-1}.x.y$, $\forall x, y \in N$ and $n \geq 2$

Proof: Trivial.

Theorem 5: A cancellative semi near ring N with identity is commutative if and only if $(x.y).x = (y.x)x \quad \forall x, y \in N$.

Proof: First part is trivial.

For the converse replace the element x with x + e in the condition. We get,

$$\left[(x+e).y\right].(x+e) = \left[y.(x+e)\right].(x+e)$$
$$\Rightarrow (x.y+y).(x+e) = (y.x+y).(x+e)$$
$$\Rightarrow (x.y).x + x.y + y.x + y = (y.x).x + y.x + y.x + y$$
$$\Rightarrow x.y = y.x \qquad \forall x, y \in N$$

Hence N is commutative.

Theorem 6: A cancellative semi near ring N with identity is commutative if and only if $(x.y)^2 = y.x^2.y, \forall x, y \in N$.

Proof: First part is trivial.

For the converse replace the element y with y + e in the given condition. We get,

$$x.(y+e).x(y+e) = (y+e).x^2.(y+e)$$
$$\Rightarrow (x.y+x).(x.y+x) = (y.x^2+x^2).(y+e)$$
$$\Rightarrow (x.y)^2 + (x.y).x + x^2.y + x^2 = y.x^2.y + (y.x).x + x^2.y + x^2$$
$$\Rightarrow (x.y).x = (y.x).x \qquad \forall x, y \in N$$

The commutativity of N follows from Theorem (5).

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